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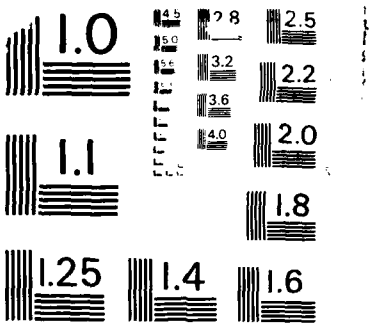

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The focus of this research is the filtering jump processes. To investigate the filtering of manifold-valued processes, their approximation by random walks and Markov chains was studied. The object was to approximate a signal process by a finite-state jump process for which a finite-dimensional filter is available. Four papers were published during the past year, including "The existence of smooth densities for the prediction, filtering and smoothing problems" and "The partially observed stochastic minimum principle".

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Martingale Representation and the Malliavin Calculus

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1. INTRODUCTION.

Using the theory of stochastic flows the integrand in a stochastic integral is identified. After some rearrangement this integrand is itself written in terms of a martingale which can be expressed as a stochastic integral, and by recursively repeating the representation a homogeneous chaos expansion is obtained. Using the stochastic integral representation an integration by parts formula is then derived. If the inverse of the Malliavin matrix M belongs to all the spaces $L^p(\Omega)$ we show a random variable has a smooth density. The difficult questions concerning the relationship between Hörmander's conditions on the coefficient vector fields and the integrability of M^{-1} are not discussed, but, at least for Markov flows, the discussion below appears to be an elementary treatment of some ideas of the Malliavin calculus. This paper was presented at the Workshop on Diffusion Approximations held at the International Institute for Applied Systems Analysis, Laxenburg, Austria, in July 1987. A fuller treatment of the ideas given here can be found in [1].

2. DYNAMICS.

Consider a stochastic differential system

$$dx_t = X_0(t, x_t)dt + X_i(t, x_t)dw_t^i. \tag{2.1}$$

Here $x \in R^d$, $0 \leq t \leq T$ and $w = (w^1, \ldots, w^m)$ is an m-dimensional Brownian motion on (Ω, F, P). We shall suppose the coefficient vector fields X are smooth and have bounded derivatives of all orders.

From results in [2], for example, it is known that for $0 \leq s \leq t \leq T$ and $x_s \in R^d$ there is a unique solution $\xi_{s,t}(x_s)$ of (2.1) with $\xi_{s,s}(x_s) = x_s$. Furthermore, there is a version of this solution which, almost surely, is smooth in $x_s \in R^d$.

If $x_0 \in R^d$ and $x = \xi_{0,t}(x_0)$, because the solutions of (2.1) are unique:

$$\begin{aligned} \xi_{0,T}(x_0) &= \xi_{t,T}(\xi_{0,t}(x_0)) \\ &= \xi_{t,T}(x). \end{aligned} \tag{2.2}$$

Write $D_{s,t} = \frac{\partial \xi_{s,t}}{\partial x}$ for the Jacobian of the map $x \to \xi_{s,t}(x)$. Then, differentiating (2.2)

$$D_{0,T} = D_{t,T} D_{0,t}.$$

Again, from [2] we know that D satisfies the equation

$$dD_{s,t} = \frac{\partial x_0}{\partial \xi} D_{s,t} dt + \frac{\partial X_i}{\partial \xi} D_{s,t} dw_t^i \tag{2.3}$$

with $D_{s,s} = I$, the $d \times d$ identity matrix.

Consider the matrix function $V_{s,t}$ defined by the stochastic differential equation

$$dV_{s,t} = -V_{s,t} \frac{\partial \overline{X}_0}{\partial \xi} dt - V_{s,t} \frac{\partial X_i}{\partial \xi} dw_t^i \tag{2.4}$$

with $V_{s,s} = I$. Here

$$\overline{X}_0^j = X_0^j - \frac{1}{2} \sum_{\alpha=1}^{m} \left(\frac{\partial X_\alpha^j}{\partial \xi_k} \right) X_\alpha^k.$$

Then, see [2], $d(V_{s,t} D_{s,t}) = 0$ so

$$V_{s,t} = D_{s,t}^{-1}.$$

3. MARTINGALE REPRESENTATION.

Suppose $x_0 \in R^d$ is given. Consider a smooth, bounded function c on R^d and the random variable $c(\xi_{0,T}(x_0))$. Write $\{F_t\}$ for the right continuous, complete filtration generated by $F_t = \sigma\{w_s : s \le t\}$. Because x_0 is known $\sigma\{x_s : s \le t\} \subset F_t$ and the process (x_t, w_t) is Markov. Consider the martingale

$$M_t = E[c(\xi_{0,T}(x_0)) \mid F_t].$$

Then by the martingale representation result

$$M_t = M_0 + \int_0^t \gamma_i(s) dw_s^i \tag{3.1}$$

for some predictable, square integrable process γ. However, because $\xi_{0,t}(x_0)$ is Markov, writing $x = \xi_{0,t}(x_0)$

$$\begin{aligned} M_t &= E[c(\xi_{0,T}(x_0)) \mid x] \\ &= E[c(\xi_{t,T}(x))] \\ &= E[c(\xi_{t,T}(x)) \mid F_t] \\ &= V(t,x). \end{aligned}$$

By the chain rule $c(\xi_{t,T}(x))$ is differentiable in x. Consequently $V(t,x)$ is differentiable in x. By considering the backward equation for $\xi_{t,T}(x)$ as in Kunita [3] we see $V(t,x)$ is differentiable in t. Therefore, applying the Ito differentiation rule to $V(t,x)$ with $x = \xi_{0,t}(x_0)$:

$$\begin{aligned} V(t,\xi_{0,t}(x_0)) = V(0,x_0) &+ \int_0^t \Big(\frac{\partial V}{\partial s} + LV\Big) ds \\ &+ \int_0^t \frac{\partial V}{\partial x} \cdot X_i dw_s^i. \end{aligned} \tag{3.2}$$

Here

$$L = \sum_{i=1}^{d} X_0^i \frac{\partial}{\partial x_i} + \frac{1}{2} \sum_{i,j=1}^{d} \Big(\sum_{k=1}^{m} X_k^i X_k^j \Big) \frac{\partial^2}{\partial x_i \partial x_j}.$$

However, $V(t,\xi_{0,t}(x_0)) = M_t$ so the decompositions (3.1) and (3.2) must be the same. The bounded variation term in (3.2) is, therefore, zero, i.e.:

$$\frac{\partial V}{\partial s} + LV = 0$$

and (as is well known) V is the solution of the backward Kolmogorov equation with a final condition

$$c(x_T) = V(T, x_T).$$

Equating the martingale terms in (3.1) and (3.2)

$$\gamma_i(t) = \frac{\partial V}{\partial x} \cdot X_i.$$

Differentiating inside the expectation

$$\frac{\partial V}{\partial x} = E[c_\xi(\xi_{t,T}(x))D_{t,T} \mid F_t]$$

by the chain rule,

$$= E[c_\xi(\xi_{0,T}(x_0))D_{0,T} \mid F_t]D_{0,t}^{-1}.$$

So

$$\gamma_i(t) = E[c_\xi(\xi_{0,T}(x_0))D_{0,T} \mid F_t]D_{0,t}^{-1}X_i \tag{3.3}$$

and

$$M_t = E[c(\xi_{0,T}(x_0))] + \int_0^t E[c_\xi(\xi_{0,T}(x_0))D_{0,T} \mid F_s]D_{0,s}^{-1}X_i(s,\xi_{0,s}(x_0))dw_s^i. \tag{3.4}$$

REMARKS 3.1. Note the term $E[c_\xi(\xi_{0,T}(x_0))D_{0,T} \mid F_s]$ is itself a martingale. If the representation is written down at $t = T$

$$M_T = c(\xi_{0,T}(x_0)) = E[c(\xi_{0,T}(x_0))] + \int_0^T E[c_\xi(\xi_{0,T}(x_0))D_{0,T} \mid F_s]D_{0,s}^{-1}X_i dw_s^i. \tag{3.5}$$

Also, the representation (3.4) holds for vector (or matrix) functions c.

If we take $c(\xi) = \xi$ to be the identity map on R^d (3.5) gives

$$\xi_{0,T}(x_0) = E[\xi_{0,T}(x_0)] + \int_0^T E[D_{0,T} \mid F_s]D_{0,s}^{-1}X_i dw_s^i.$$

Also, if we consider (3.5) for a second smooth bounded function g and take the expected value of the product of each side, we see:

$$E[c(\xi_{0,T}(x_0))g(\xi_{0,T}(x_0))] = E[c(\xi_{0,T}(x_0))]E[g(\xi_{0,T}(x_0))]$$

$$+ E\Big[\sum_{i=1}^m \int_0^T E[c_\xi D_{0,T} \mid F_s]D_{0,s}^{-1}X_i X_i^* D_{0,s}^{*-1} E[g_\xi D_{0,T} \mid F_s]^* ds\Big]. \tag{3.6}$$

DEFINITION 3.2. *The Malliavin matrix for the system (2.1) is*

$$M_{s,t} = \sum_{i=1}^m \Big(\int_s^t D_{s,u}^{-1}X_i(u)X_i^*(u)D_{s,u}^{*-1}du\Big).$$

Note something resembling $M_{0,s}$ occurs in (3.6).

4. HOMOGENEOUS CHAOS EXPANSIONS.

Consider an enlarged system with components $\xi^{(1)} = (\xi, D)$. The stochastic differential equation for $\xi^{(1)}$ is, therefore, the system (2.1) and (2.3). The coefficients in (2.3) are no longer bounded, but following Norris [4] a sequence of 'triangular' systems can be considered and the results on stochastic flows still hold. We can, therefore, consider the Jacobian $D^{(1)}$ of the system $\xi^{(1)}$ and a system $\xi^{(2)} = (\xi^{(1)}, D^{(1)})$. Proceeding in this way $\xi^{(n)}$ is a system with components $(\xi^{(n-1)}, D^{(n-1)})$. Write

$$c^{(1)} = \frac{\partial c}{\partial \xi} D_{0,T}$$

$$c^{(2)} = \frac{\partial c^{(1)}}{\partial \xi^{(1)}} D^{(1)}_{0,T} \qquad \text{etc.}$$

Equation (3.4) can then be written

$$c(\xi_{0,T}(x_0)) = E[c(\xi_{0,T}(x_0))] + \int_0^T E[c^{(1)} \mid F_s] D^{-1}_{0,s} X_i dw^i_s. \tag{4.1}$$

However, $E[c^{(1)}(\xi^{(1)}_{0,T}) \mid F_s]$ can be represented, as in Section 3, as a stochastic integral

$$E[c^{(1)} \mid F_s] = E[c^{(1)}] + \int_0^s E[c^{(2)} \mid F_{s_1}] D^{(1)-1}_{0,s_1} X^{(1)}_j(s_1) dw^j_{s_1}.$$

Here, $X^{(1)}_j$ is the coefficient vector field of w^j in the system defining $\xi^{(1)}$. Substituting in (4.1)

$$\begin{aligned} c(\xi_{0,T}(x_0)) = E[c] + E[c^{(1)}] \int_0^T D^{-1}_{0,s} X_i dw^i_s \\ + \int_0^T \Big(\int_0^s E[c^{(2)} \mid F_{s_1}] D^{(1)-1}_{0,s_1} X^{(1)}_j(s_1) dw^j_{s_1} \Big) D^{-1}_{0,s} X_i dw^i_s. \end{aligned} \tag{4.2}$$

Now $E[c^{(2)} \mid F_{s_1}]$ can be expressed as a stochastic integral and the result substituted in (4.2). Proceeding in this way we obtain the homogeneous chaos expansion of the random variable $c(\xi_{0,T}(x_0))$. The repeated stochastic integrals do not involve c but only the Jacobians $D^{(k)}$ and coefficients $X^{(k)}$.

5. INTEGRATION BY PARTS.

LEMMA 5.1. *Suppose $u = (u_1, \ldots, u_m)$ is a square integrable predictable process. Then*

$$E[c(\xi_{0,T}(x_0)) \int_0^T u_i dw_s^i] = \sum_{i=1}^m E\Big[c_\xi(\xi_{0,T}(x_0)) D_{0,T} \int_0^T D_{0,s}^{-1} X_i(s) u_i(s) ds\Big].$$

PROOF. Consider the representation (3.5) for $c(\xi_{0,T}(x_0))$. Multiply by $\int_0^T u_i dw_s^i$ and, using Fubini's theorem, take the expectation.

COROLLARY 5.2. *Take $u_i(s) = (D_{0,s}^{-1} X_i(s))^*$. Then*

$$E[c(\xi_{0,T}(x_0)) \int_0^T (D_{0,s}^{-1} X_i(s))^* dw_s^i] = E[c_\xi(\xi_{0,T}(x_0)) D_{0,T} M_{0,T}]. \tag{5.1}$$

REMARKS 5.3. Consider a product function $h(\xi_{0,T}(x_0)) = c(\xi_{0,T}(x_0)) g(\xi_{0,T}(x_0))$ and apply Corollary 5.2 to h. Then

$$E[(cg)(\xi_{0,T}(x_0)) \int_0^T (D_{0,s}^{-1} X_i)^* dw_s^i] = E[(c_\xi g + c g_\xi) D_{0,T} M_{0,T}]. \tag{5.2}$$

We would like to take $g = M_{0,T}^{-1} D_{0,T}^{-1}$ in (5.2) so that we can obtain a bound for c_ξ. This can be done by considering, again following Norris [4], a hierarchy of stochastic systems similar to, but different from, those introduced in Section 4.

This time write $\phi^{(0)}(w, s, t, x) = \xi_{s,t}(x)$ for the flow defined by (2.1) and $D_{s,t}^{(0)}(x) = D_{s,t}(x)$ for its Jacobian. $R_{s,t}^{(0)} = \int_s^t (D_{s,u}^{-1} X_i(u))^* dw_u^i$ and $M_{s,t}^{(0)} = M_{s,t}$ is the Malliavin matrix defined in (3.2). Note that $M_{s,t}$ can be considered as the predictable quadratic variation of the tensor product of $R^{(0)}$ with its adjoint that is $M_{s,t}^{(0)} = \langle R^{(0)} \otimes R^{(0)^*} \rangle_{s,t}$.

Now consider an enlarged system $\phi^{(1)}$ with components

$$\phi^{(1)} = (\phi^{(0)},\ D^{(0)},\ R^{(0)},\ M^{(0)}).$$

The results of Norris [4] on stochastic flows allow us to discuss the Jacobian $D^{(1)}$ of $\phi^{(1)}$. Suppose $X_i^{(1)}$ is the coefficient of w^i in the system describing $\phi^{(1)}$, and write

$$R_{s,t}^{(1)} = \int_s^t (D_{s,u}^{(1)^{-1}} X_i^{(1)}(u))^* dw_u^i$$

$$M_{s,t}^{(1)} = \langle R^{(1)} \otimes R^{(0)^*} \rangle_{s,t}.$$

Then define

$$\phi^{(2)} = (\phi^{(1)},\ D^{(1)},\ R^{(1)},\ M^{(1)})$$

and inductively, $\phi^{(n+1)} = (\phi^{(n)},\ D^{(n)},\ R^{(n)},\ M^{(n)})$. Write ∇_n for the gradient operator in the components of $\phi^{(n)}$. The following result is established like equation (5.2) by considering the martingale representation (3.5) of the produce cg.

THEOREM 5.4. *Suppose c is a bounded C^∞ scalar function on R^d with bounded derivatives. Let g be a C^∞ possibly vector, or matrix, valued function on the state space of $\phi^{(n)}$ such that $g(\phi^{(n)}(0,T,x_0))$ and $\nabla_n g(\phi^{(n)}(0,T,x_0))$ are both in some $L^p(\Omega)$. Then*

$$\begin{aligned} &E[c(\phi^{(0)}(0,T))g(\phi^{(n)}(0,T)) \otimes R^{(0)}_{0,T}] \\ &\quad = E[(\nabla_0 c)(\phi^{(0)}(0,T))g(\phi^{(n)}(0,T))D_{0,T}\,M_{0,T}] \\ &\qquad + E[c(\phi^{(0)}(0,T))(\nabla_n g)(\phi^{(n)}(0,T))D^{(n)}_{0,T}\,M^{(n)}_{0,T}]. \end{aligned} \tag{5.3}$$

COROLLARY 5.5. *Gronwall's inequality shows that D^{-1} is in all the $L^p(\Omega)$ spaces, so if $M^{-1}_{0,T}$ is in some $L^p(\Omega)$ taking $g(\phi^{(1)}(0,T)) = M^{-1}_{0,T}\,D^{-1}_{0,T}$ in (5.3)*

$$\begin{aligned} E[c_\xi(\xi_{0,T}(x_0))] &= E[c(\xi_{0,T}(x_0))M^{-1}_{0,T}\,D^{-1}_{0,T} \otimes R_{0,T}] \\ &\quad - E[c(\xi_{0,T}(x_0))(\nabla_1 g)(D_{0,T}, M_{0,T})D^{(1)}_{0,T}\,M^{(1)}_{0,T}]. \end{aligned}$$

Because c is bounded we, therefore, have the following result:

THEOREM 5.6. *Suppose $\xi_{0,T}(x_0)$ is the solution of (2.1) and c is any smooth bounded function with bounded derivatives. Then if $M^{-1}_{0,T}$ is in some $L^p(\Omega)$*

$$|E[c_\xi(\xi_{0,T}(x_0))]| \le K \sup_{x \in R^d} |c(x)|. \tag{5.4}$$

REMARKS 5.6. It is well known that (4.3) implies the random variable $\xi_{0,T}(x_0)$ has a density $d(x)$. To show the density d is smooth we wish to establish inequalities of the

form

$$\left|E\left[\frac{\partial^\alpha c}{\partial \xi^\alpha}(\xi_{0,T}(x_0))\right]\right| \leq K \sup_{x \in R^d} |c(x)|. \tag{5.5}$$

Here

$$\frac{\partial^\alpha}{\partial \xi^\alpha} = \frac{\partial^{\alpha_1}}{\partial \xi_1^{\alpha_1}} \frac{\partial^{\alpha_2}}{\partial \xi_2^{\alpha_2}} \cdots \frac{\partial^{\alpha_d}}{\partial \xi_d^{\alpha_d}}.$$

An argument from Fourier analysis, (see [4]), shows that if (5.5) is true for all α with $|\alpha| = \alpha_1 + \cdots + \alpha_d \leq n$ where $n \geq d+1$ then the random variable $\xi_{0,T}(x_0)$ has a density $d(x)$ which is in $C^{n-d-1}(R^d)$.

Apply Corollary 5.5 to c_ξ rather than c so

$$E[c_{\xi\xi}(\xi_{0,T}(x_0))] = E[c_\xi(\xi_{0,T}(x_0))M_{0,T}^{-1}D_{0,T}^{-1} \otimes R_{0,T}]$$
$$- E[c_\xi(\xi_{0,T}(x_0))(\nabla_1 g)(D_{0,T}, M_{0,T})D_{0,T}^{(1)}M_{0,T}^{(1)}]. \tag{5.6}$$

Consider the two terms on the right of (5.6) and write $M = M_{0,T}$, $D = D_{0,T}$, etc. Let

$$g_1(\phi^{(1)}) = M^{-1}D^{-1} \otimes RM^{-1}D^{-1}$$

and $\quad g_2(\phi^{(2)}) = (\nabla_1 g)(D, M)D^{(1)}M^{(1)}M^{-1}D^{-1}.$

Applying Theorem 5.4 to cg_1 and cg_2:

$$E[c(\xi_{0,T}(x_0))g_1(\phi^{(1)}) \otimes R] = E[c_\xi(\xi_{0,T}(x_0))M^{-1}D^{-1} \otimes R]$$
$$+ E[c(\xi_{0,T}(x_0))(\nabla_2 g_1)(\phi^{(2)})D^{(2)}M^{(2)}]. \tag{5.7}$$

and

$$E[c(\xi_{0,T}(x_0))g_2(\phi^{(2)}) \otimes R] = E[c_\xi(\xi_{0,T}(x_0))(\nabla_1 g)(D, M)D^{(1)}M^{(1)}]$$
$$+ E[c(\xi_{0,T}(x_0))(\nabla_3 g_2)(\phi^{(3)})D^{(3)}M^{(3)}]. \tag{5.8}$$

Using (5.7) and (5.8) the terms on the right of (5.6) can be replaced by terms involving c. This procedure can be iterated using Theorem 5.4 and the following result established:

THEOREM 5.7. *Suppose* M^{-1} *is in all spaces* $L^p(\Omega)$, $1 \leq p < \infty$. *Then the random variable* $\xi_{0,T}(x_0)$ *has a smooth density.*

The remaining questions concern the existence and integrability properties of $M_{0,T}^{-1}$. These have been carefully studied; see Ikeda and Watanabe [2], or Norris [4], for example. In fact $M_{0,T}^{-1}$ is in $L^p(\Omega)$ for all p, $1 \leq p < \infty$, if the following condition of Hörmander is satisfied:

CONDITION 5.8. The vector space $V(x_0)$ generated by the coefficient vector fields $X_1, \ldots, X_m$ and the brackets $[X_i, X_j]$, $0 \leq i, j \leq m$, $[X_i, [X_j, X_k]]$, $0 \leq i, j, k \leq m$ etc., evaluated at $x_0 \in R^d$, is the whole of R^d.

REFERENCES

1. R.J. Elliott and M. Kohlmann. *Integration by parts, homogeneous chaos expansions and smooth densities.* University of Alberta Technical Report, 1987.
2. N. Ikeda and S. Watanabe. *Stochastic differential equations and diffusion processes.* North Holland Publishing Co., Amsterdam, Oxford, New York, 1981.
3. H. Kunita. *Stochastic partial differential equations connected with non-linear filtering.* Lecture Notes in Math, 972 (1982).
4. J. Norris. *Simplified Malliavin calculus.* Lecture Notes in Math, 1204 (1986).

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